

shock test are equal, the peak accelerations of those two tests should yield the same theoretical result. However, experimental data shows that for such tests with equal drop heights, the peak accelerations are certainly not equal. The prime reason is that the ubiquitous phenomenon of friction in the real environment is not considered under the assumption mentioned above.

2. MATERIALS AND METHODS

2.1 Traditional equivalent drop theory

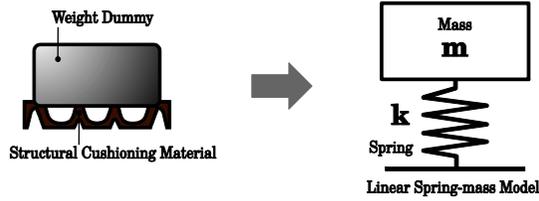


Fig. 1. Traditional Model

The structural cushioning materials are assumed to follow a linear spring-mass model under the traditional equivalent drop theory (traditional theory in this paper) (Fig. 1). In this assumption, the peak acceleration occurring on a weight dummy $A_{ff \max}$ of the dynamic compression test is shown as follows:

$$A_{ff \max} = \omega_n \cdot V \quad (1)$$

$$\because \omega_n = \sqrt{k/m} \quad V = \sqrt{2gh},$$

where ω_n is the natural angular frequency with no attenuation, m is the mass, k is the spring constant, h is the drop height, and V is the velocity of the weight dummy.

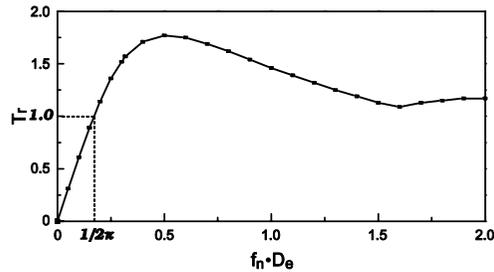


Fig. 2. Shock response spectrum (SRS)

For the shock test, when an extremely short half-sine shock pulse is applied to the system, the shock response spectrum (SRS) of the system is recorded, as shown in Fig. 2. When $f_n \cdot D_e$ (where f_n is the natural frequency with no attenuation, and D_e is the effective impact duration) is smaller than $1/2\pi$, the model is regarded as a "soft spring" and the SRS curve can be expressed as linear with slope 2π . Hence, the shock transmissibility T_r is equal to $2\pi f_n D_e$, and T_r can be represented as follows^[3]:

$$T_r = \frac{A_{cs \max}}{A_{in}} = 2\pi \cdot f_n \cdot D_e \tag{2}$$

$$A_{cs \max} = \omega_n \cdot V_c \tag{3}$$

$$\therefore \omega_n = 2\pi \cdot f_n \quad V_c = D_e \cdot A_{in} \quad D_e = 2D/\pi,$$

where $A_{cs \max}$ is the peak acceleration, A_{in} is the input acceleration, V_c is the velocity change on the shock table, and D is the impact duration.

Comparing **Eqs.(1)** and **(3)**, if we let $V = V_c$ when we perform the dynamic compression test and the shock test, the peak accelerations of those two tests should be the same theoretically. In other words, we can draw the conclusion that the two tests are equivalent.

2.2 Tests based on the traditional theory

First of all, the dynamic compression test and the shock test, which are based on the traditional equivalent drop theory, are performed.

2.2.1 Decision of equivalent free-fall height

According to JIS-Z-0240-2002, equivalent free-fall height should be set at 60 cm, which is adopted as described in **Section. 2.1**.

2.2.2 Low-pass filtering

In order to remove noise from the target pulse and the inherent vibration of the weight dummy, low-pass filtering must be correctly set up. According to JIS-Z-0240-2002, 300 or 500 Hz low-pass filtering can often be employed in a test. With respect to structural cushioning materials, the acceleration may become very complicated due to their different shape. Thus, 300 Hz low-pass filtering is recommended for the test of structural cushioning materials.

2.2.3 Test equipment and materials

The test equipment and materials used in this paper are shown as follows:

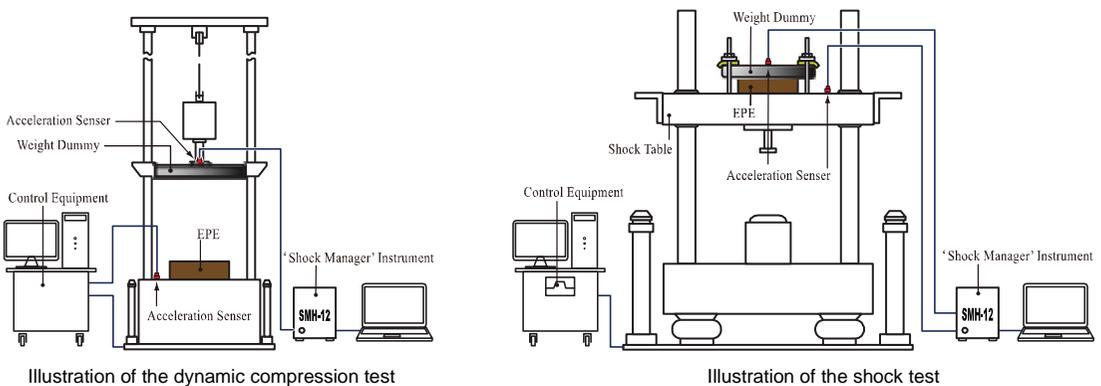


Fig. 3. Illustration of the dynamic compression test and the shock test

- 1) Equipment (Fig. 3):
 - ◆ Dynamic compression tester
 - ◆ Shock machine
 - ◆ "Shock manager" measuring and test instrument.
- 2) Test materials:
 - ◆ Weight dummy (Fig. 3)
[Mass: 4kg]
 - ◆ Expanded polyethylene from Asahi Kasei Chemicals Corporation (Fig. 4)
[Expansion rate: 25, Size: 100 × 100 × 30 mm]

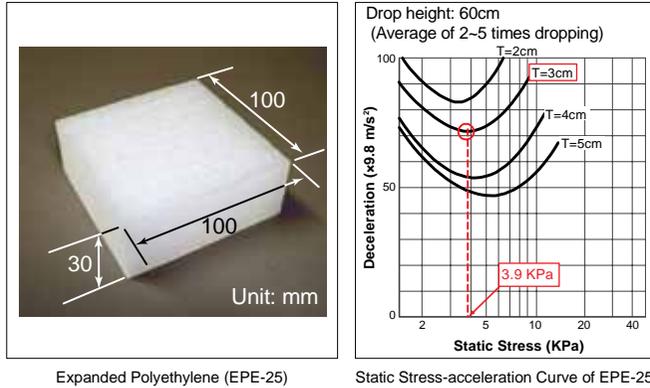


Fig. 4. Expanded polyethylene (EPE-25)

The weight dummy is selected using the following procedure: As shown in Fig. 4, according to the thickness of the test material and the official static stress-peak acceleration curve, when the drop height is 60 cm, we draw a line from the lowest point of the curve described as $T = 3 \text{ cm}$ to the x axis. Then we find the static stress acting on the material to be 3.9 KPa. It is easy to calculate that the mass of the weight dummy is 4 kg with the formula $\sigma = P/A$, where σ is the static stress, P is the load, and A is the compression area.

2.2.4 Setup of the test condition

The test materials should be placed in a controlled atmospheric condition where the temperature is 23 and the relative humidity is 50% for 24 hours before the dynamic compression test and the shock test.

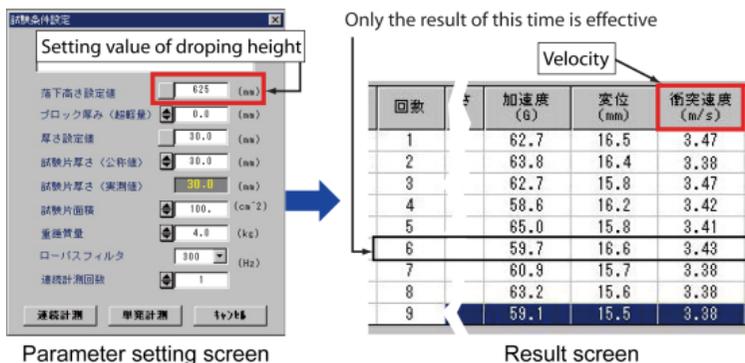


Fig. 5. Parameter setting in the dynamic compression test

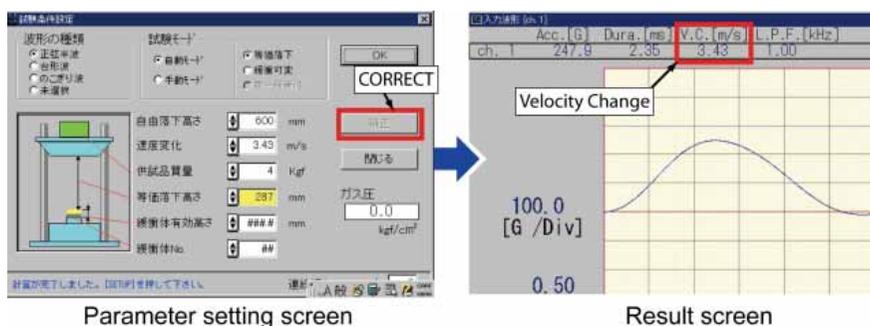


Fig. 6. Parameter setting in the shock test

The mass of the weight dummies used in both tests is the same (4 kg), and an acceleration sensor used for measuring the peak acceleration is fixed in the center above the weight dummy.

Because the equivalent free-fall height is 60 cm, we can calculate that the theoretical velocity is 3.43 m/s according to the formula $V = \sqrt{2gh}$. The parameter of the dynamic compression test and the shock test are set by the test control software separately, as shown in Figs. 5 and 6.

When the dynamic compression test is performed, the set-up drop height is not the same as the equivalent free-fall height (60 cm), but is equal to the corresponding drop height for the real velocity of the weight dummy to the same as the theoretical velocity (3.43 m/s). Therefore, the test has to be adjusted repeatedly until the velocity of the weight dummy becomes 3.43 m/s.

On the other hand, when the shock test is performed, in order to adjust the velocity change of the shock waveform occurring on the shock table to 3.43 m/s, an adjusting function in the controlling software (Fig. 6) needs to be used by repetition, too.

For two tests, the results are effective only when $V = 3.43 \text{ m/s}$ or $V_c = 3.43 \text{ m/s}$.

2.2.5 Result of two tests

For both tests, the effective results are shown in Fig. 7.

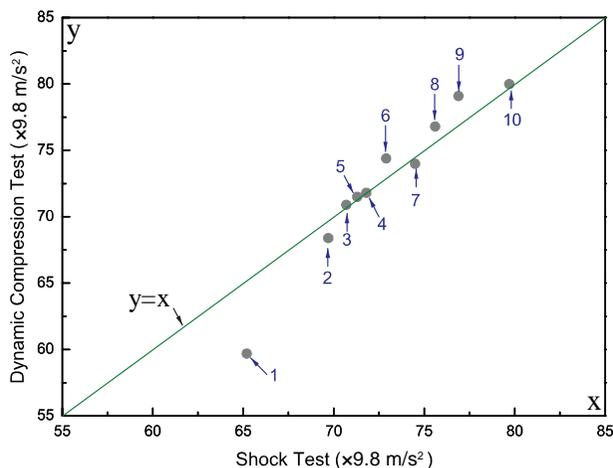


Fig. 7. Result of two tests based on the traditional theory

As the expanded polyethylene's cushioning performance decreases as the test proceeds, the peak accelerations of the two tests gradually increase and there is a linear relationship between the accelerations observed under the two tests. the result shown in Fig. 7 is not the same as a theoretical result that all the equivalent points of the two tests should be exactly on the line $y = x$ under the traditional theory, but a majority of them are scattered around the line. This establishes the limitation of the traditional equivalent drop theory.

2.3 Modified equivalent drop theory^[3]

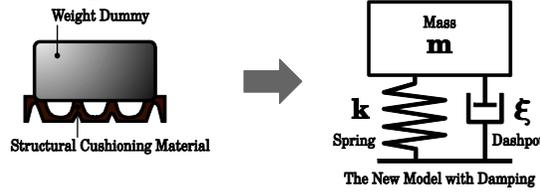


Fig. 8. Attenuation Model

First, the structural cushioning material is assumed to obey a one-dimensional attenuation model (Fig. 8), and the peak acceleration occurring on the weight dummy $A_{ff \max}$ of the dynamic compression test is shown as follows:

$$A_{ff \max} = n_{ff} \cdot \omega_n \cdot V \quad (4)$$

$$n_{ff} = \left| \frac{2\xi^2 - 1}{\sqrt{1 - \xi^2}} \right| \cdot \exp\left(\frac{-\pi\xi}{2\sqrt{1 - \xi^2}}\right) \quad (5)$$

where n_{ff} is the correction coefficient of the dynamic compression test, and ξ is the damping ratio.

For the shock test, the response acceleration of the attenuation model in which input acceleration is the half-sine wave can be obtained theoretically. Subsequently, T_r can be calculated using the input and response acceleration. If we define n_{cs} as the ratio between T_r and the approximation of the shock transmissibility based on traditional theory $2\pi f_n D_e$, we can obtain

$$n_{cs} = \frac{T_r}{2\pi \cdot f_n \cdot D_e}, \quad (6)$$

then the peak acceleration of the shock test $A_{cs \max}$ can be as follows:

$$A_{cs \max} = n_{cs} \cdot \omega_n \cdot V_c \quad (7)$$

where n_{cs} is the correction coefficient of the shock test, and V_c is the velocity change on the shock table.

With respect to the preceding information, a condition---*correction coefficient*---must be considered if we want the dynamic compression test and the shock test to be equivalent under the attenuation model, and the expression of the specific condition is

$$V_c = \frac{n_{ff}}{n_{cs}} V. \quad (8)$$

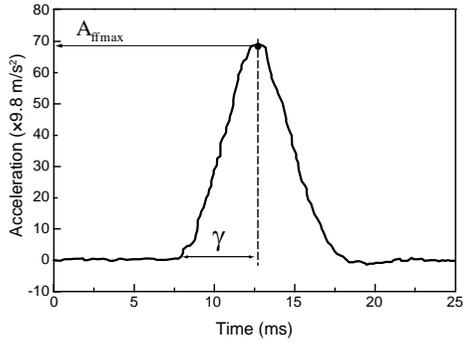


Fig. 9. Illustration of parameter γ

It must be pointed out that the attenuation model's f_n and ξ must be known in order to be able to calculate the velocity change correction coefficient. These two parameters can be obtained from the time-acceleration curve that contains the peak acceleration $A_{ff\ max}$ and the half-duration from 0 to $A_{ff\ max}$ of the shock pulse γ (Fig. 9). The method mentioned above is the correct shock test procedure for deriving the velocity change correction coefficient based on the cushioning materials' dynamic compression test.

2.4 Correction of the velocity change

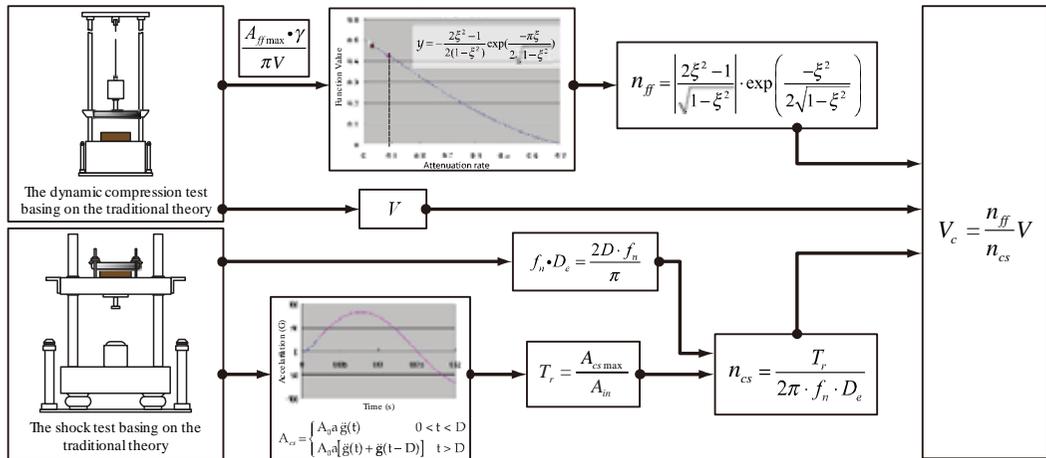


Fig. 10. Flowchart for correcting the velocity change V_c

Under the new equivalent drop theory, a correction of the velocity change follows the flowchart in Fig. 10.

We can get the response acceleration-time curve after the dynamic compression test is performed (Fig. 9), and γ can be expressed with ξ and ω_n as:

$$\gamma = \frac{\pi}{2\omega_n \sqrt{1 - \xi^2}} \tag{9}$$

then, we combine **Eq.(9)** with **Eqs.(4)** and **(5)**, **Eq.(10)** is derived:

$$\frac{A_{ff \max} \cdot \gamma}{\pi V} = \pm \frac{2\xi^2 - 1}{2(1 - \xi^2)} \exp\left(\frac{-\pi\xi}{2\sqrt{1 - \xi^2}}\right) \quad (10)$$

where "-" is valid for $\xi < 0.707$, and "+" for $\xi > 0.707$.

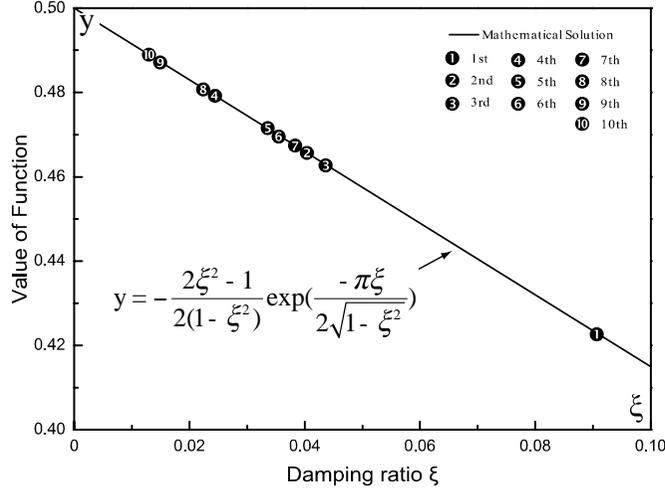


Fig. 11. Calculation of the damping ratio ξ

For the right-hand side of **Eq.(10)**, a curve that relates to ξ can be plotted (Fig. 11). Furthermore, a specific value can be calculated if the metadata of the dynamic compression test is imported into the left-hand side of **Eq.(10)**. Next we match these specific values with the curve graphics; the corresponding damping ratio ξ is available when both of them match. At last, n_{ff} can be calculated after ξ is imported into **Eq.(5)**.

On the other hand, the response acceleration of the shock test can be expressed as follows:

$$A_{cs}(t) = \begin{cases} A_0 a \cdot \ddot{g}(t) & 0 < t < D \\ A_0 a \cdot [\ddot{g}(t) + \ddot{g}(t - D)] & t > D \end{cases},$$

$$\therefore \ddot{g}(t) = \frac{(2a^2\omega^2 - 4a^2\omega^2\xi^2 - 2\omega^4) \sin(\omega\sqrt{1 - \xi^2} \cdot t) + 4a^2\omega^2\xi\sqrt{1 - \xi^2} \cos(\omega\sqrt{1 - \xi^2} \cdot t)}{2\omega\sqrt{1 - \xi^2} [\omega^4 + (4\xi^2 - 2)a^2\xi^2 + a^4]} \exp(-\omega\xi t) \quad (11)$$

$$+ \frac{(4a^2\omega^2\xi^2 + \omega^4 - a^2\omega^2) \sin(at) - 2a^3\omega\xi \cos(at)}{a [\omega^4 + (4\xi^2 - 2)a^2\omega^2 + a^4]},$$

$$a = \pi/T_0,$$

where T_0 is the initial impact duration. The maximum value of the response acceleration can be determined by means of the shock response analysis (SR analysis)^[4] (Fig. 12), and then T_r can be calculated using **Eq.(2)**. The corrected velocity change V_c can eventually be calculated according to **Eq.(8)**.

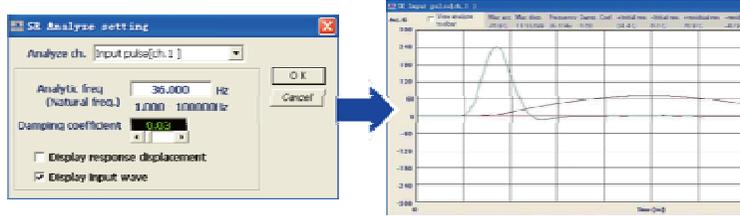


Fig. 12. SR analysis

Using the corrected velocity change V_c , the shock test is re-performed, and the new test data are obtained. The comparison of the result based on the traditional theory and the result after being corrected by *the correcting method-1* (The method in which the input velocity change of the shock test is determined after solving the model's f_n and ξ according to the dynamic compression test is called the correcting method-1 in this paper.) is shown in Fig. 13.

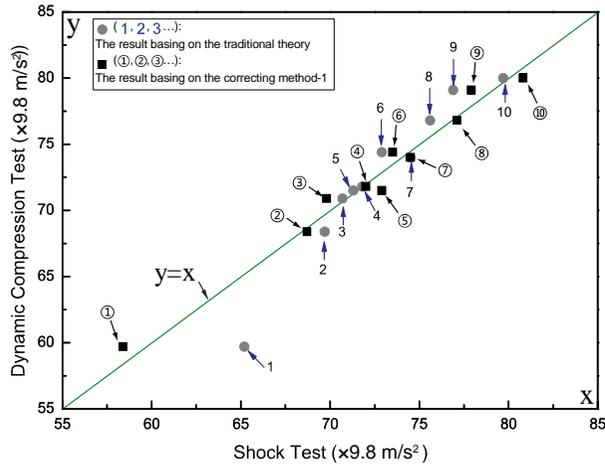


Fig. 13. Comparison of results of the tests based on the traditional method and the correcting method-1

2.5 Equivalent drop height

The equivalent drop height can also be determined according to the shock test. First, the frequency is calculated using the response acceleration waveform of the weight dummy during the shock test. Second, SR analysis is performed and the desired damping ratio is obtained. n_{cs} can be determined after T_r is calculated using the input and response acceleration. Next, n_{ff} is obtained using **Eq.(5)**. Finally, the equivalent drop height corresponding with the free-fall test can be calculated according to the following equation:

$$\begin{aligned}
 h &= \frac{V^2}{2g} = \frac{1}{2g} \left(\frac{n_{cs}}{n_{ff}} V_c \right)^2 \\
 \therefore V &= \sqrt{2gh} = \frac{n_{cs}}{n_{ff}} V_c
 \end{aligned}
 \tag{12}$$

2.6 Correction of the velocity

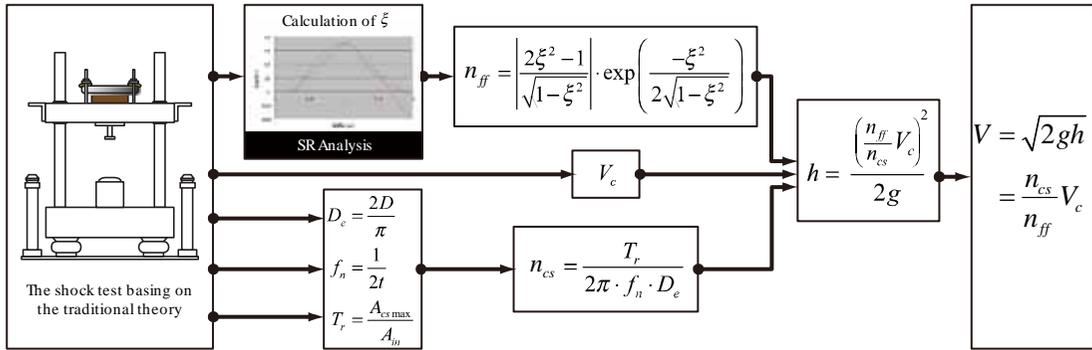


Fig. 14. Flowchart for correcting the velocity V

Fig. 14 shows the flowchart for correcting the velocity. First, the shock test based on the traditional theory is performed, and the input and response accelerations are measured. The reciprocal number of 2 times the period of a half cycle in the response acceleration is the model's natural frequency f_n . Subsequently, SR analysis is performed using the data of the input acceleration to compute the desired damping ratio. T_r can be calculated using the input acceleration and the response acceleration, and n_{cs} is determined after T_r is imported into Eq.(6), subsequently, n_{ff} is obtained when we use Eq.(5). Finally, the corrected velocity V can be calculated according to Eq.(12).

Using the corrected velocity V , the dynamic compression test is re-performed, and the new test data are obtained. The comparison of the result based on the traditional theory and the result after being corrected by *the correcting method-2* (the method in which the drop height of the dynamic compression test is determined after solving the model's f_n and ξ according to the shock test is called the correcting method-2 in this paper.) is shown in Fig. 15.

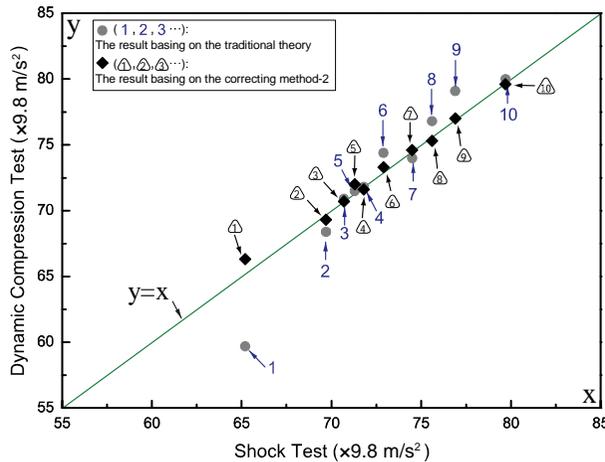


Fig. 15. Comparison of results of the tests based on the traditional method and the correcting method-2

3. RESULTS AND DISCUSSION

3.1 Correlation Coefficient

In statistics, correlation and dependence are two of a broad class of statistical relationships between two or more random variables or observed data values. Correlations are useful to indicate a predictive relationship that can be exploited in practice. There are several correlation coefficients, but the sample correlation coefficient, denoted ρ , which is mainly sensitive to a linear relationship between two variables, is in common use. The sample correlation coefficient ρ between two random variables X and Y with expected values μ_X and μ_Y and standard deviations σ_X and σ_Y is defined as

$$\rho = \frac{E((X - \mu_X)(Y - \mu_Y))}{\sigma_X \sigma_Y}, \tag{13}$$

where E is the expected value operator.

The value of the correlation coefficient must satisfy the inequality $-1 \leq \rho \leq 1$, indicating the degree of linear dependence between the variables. The closer the coefficient is to either -1 or 1 , the stronger the correlation between the variables.^[5]

3.2 Discussion of results

Table 1. Comparison of the cushioning material evaluation in the peak acceleration (G-factor)

Order	Traditional Theory		Modified Theory	
	DC Test ^a (G)	Shock Test (G)	Shock Test (G)	DC Test (G)
1	59.70	65.20	58.40	66.30
2	68.40	69.70	68.70	69.30
3	70.90	70.70	69.80	70.70
4	71.80	71.80	72.00	71.60
5	71.50	71.30	72.90	72.00
6	74.40	72.90	73.50	73.30
7	74.00	74.50	74.50	74.60
8	76.80	75.60	77.10	75.30
9	79.10	76.90	77.90	77.00
10	80.00	79.70	80.80	79.60

^a The dynamic compression test.

The values in Table 1 are the peak accelerations (G-factor) of the weight dummy. The values from the left to the right are: the result of the dynamic compression test when the weight dummy's velocity is equal to the velocity in which the corresponding drop height is 60 cm, the result of the shock test that uses the half-sine wave based on the traditional theory (the velocity change determined by the half-sine wave equals the velocity of the weight dummy), the result of the shock test after being corrected by the correcting method-1, and the result of the dynamic compression test after being corrected by the correcting method-2.

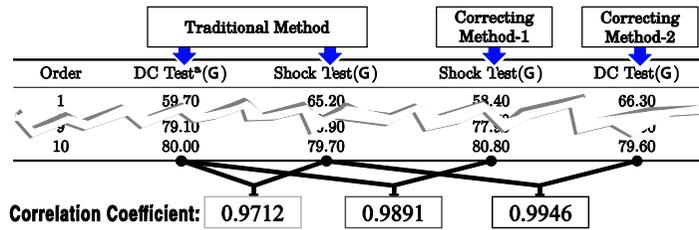


Fig. 16. Comparison of the correlation coefficient

Between the above data, three comparisons (Fig. 16) are performed to calculate the correlation coefficient, and the result is shown in Fig. 16. It is easy to see that the correlation relationship between the results becomes gradually stronger from the traditional method to the correcting method-1 to the correcting method-2. Therefore, the equivalent precision of the tests based on the modified theory is increased over those based on the traditional theory, and we can see that the correcting method-2 is superior to the correcting method-1. The reason is considered to be as follows: For the spring-mass model, the peak acceleration of the weight dummy^[4] is

$$A_{out}(t) = \omega_n \int_0^t A_{in}(\tau) \cdot \sin(\omega_n(t - \tau)) d\tau \quad (14)$$

where t and τ denote time. For the correcting method-1, the input acceleration is assumed as the half-sine wave when the dynamic compression test is performed, whereas real pulse is applied as the input acceleration when the correcting method-2 is adopted; therefore, the response acceleration based on the actual waveform, according to **Eq.(14)**, is more accurate.

4. CONCLUSIONS

Based on the preceding analysis, the traditional theory is limited because it does not consider an effect of attenuation phenomenon. Consequently, the result of the equivalent drop test based on the traditional theory has errors. In this paper, the concept of velocity correction coefficient is introduced, and the new equivalent model considering the attenuation is established. The experiment shows that the equivalence accuracy of two tests is further improved after correction; thus, it can be seen that the velocity correction is the key to reducing the error in the equivalent drop test.

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緩衝性能確認等価落下試験の革新

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落下衝撃から包装内容品を保護するための緩衝材の性能は、動圧縮試験によって評価されるが、複雑な形状の構造体緩衝材では、動圧縮試験ができないため、衝撃試験装置を用いた等価落下試験で評価される。等価落下試験では、発生させる衝撃台の正弦半波パルスの速度変化と等しい衝突速度となる落下高さが等価自由落下高さであると規定されており、緩衝系が減衰のない線形バネ-質量1自由度系とみなせる場合には実用上問題ないが、実際の緩衝系は減衰の影響が無視できない。本論文では、緩衝材性能評価としての動圧縮試験と等価落下試験の結果が必ずしも等価とみなせるほどではないことを実験的に明らかにするとともに、減衰の影響を考慮した理論に基づいた等価落下試験の方法を提案する。さらに、減衰等価落下モデルによる等価落下試験を行い、両試験の等価度を実験的に確認する。

キーワード：等価落下試験、等価自由落下高さ、動圧縮試験、衝撃試験、修正係数