Effect of Temperature/Relative Humidity Changes on Compression Strength of Corrugated Containers for Food Products: How should it be modeled and evaluated?

Chiaki MURAO

The effect of cyclic environment on the compression strength of corrugated containers which are filled with carton boxes is investigated. The compression strength characteristics at constant temperature and relative humidity conditions are tested under various cyclic environmental conditions. These observations give that there is the suitable time accelerated effect for the real stacking life of those in cyclic environmental condition. The cyclic environmental model conditions for compression strength are suggested to evaluate which is the optimum design at constant environmental condition.

Keywords: Corrugated container, Carton box, Compression strength, Constant environmental condition, Cyclic environmental condition, Stacking life, Optimum design

1. Introduction

The compression strength characteristics of corrugated containers filled with carton boxes were studied at constant temperature and relative humidity conditions by using mathematical model in the previous papers[1)-5)].

Compression test samples were provided in advance with being conditioned at 20℃, 65%RH/40℃, 90%RH for more than 48 hr without top loads. Compression creep tests were carried out in the same constant environments of conditioning.

It is well known that the stacking life of corrugated containers is reduced by exposure to high RH. Because most warehouses do not have controlled RH environments, cyclic RH is representative of the real life situation in which corrugated containers are used.

There were several studies on the compression creep characteristics of corrugated containers made from various materials in cyclic RH environments[6)-13)]. These results indicated large varieties associated with variation in board composition. Consequently, generalized criteria for the compression creep characteristics of corrugated containers in cyclic RH environments shall not be derived from these results.

In this paper, the mathematical model and its analytical results[1)-5)] in constant environment are evaluated to match up to the broad cyclic environments, and meet the failure reports of commercially sold commodities for many years.

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2. Materials and Methods

2.1 Materials and packaging styles

Corrugated containers filled with two kinds of carton boxes are tested in this study. One is small size. The other is medium size.

Filling patterns of carton boxes in corrugated containers (Fig. 1) are vertical style and perpendicular style. Two kinds of corrugated containers are tested. One is wrap around type. The other is regular slotted type. Combinations of carton boxes and corrugated containers (Table 1) are provided to investigate the adaptability of results obtained in constant temperature and RH environments studies[2)-5)] for cyclic environments.

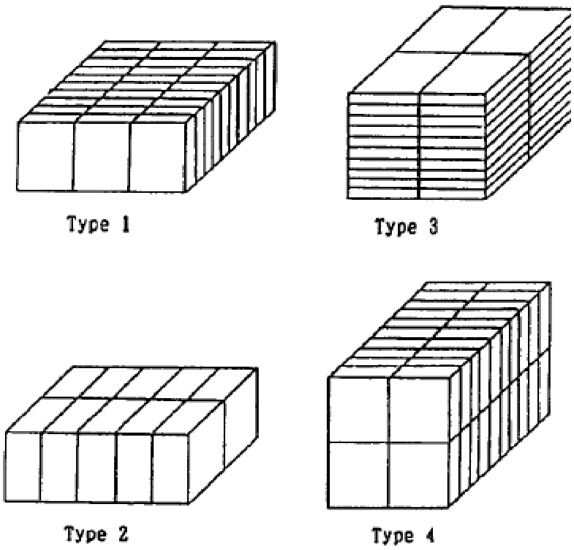

Fig.1 Filling pattern of the carton boxes in corrugated containers

Namely, model I, II and III are the packaging styles of commercially sold commodities. Model I-var. is improved on model I for cost reduction by modifying the board composition of corrugated containers associated with distributing compression loads uniformly to the carton boxes[2),3)]. Model II-var. is improved on model II by reinforcement of compression strength with optimizing top clearance[4),5)].

Model III-var.l is improved on model III by reinforcement of compression strength with modifying the filling patterns of carton boxes[2),3)]. Moreover, model III-var.2 is improved on model III-var.l for cost reduction by modifying the board composition of corrugated containers.

2.2 Methods

Cyclic temperature and RH environments are car-

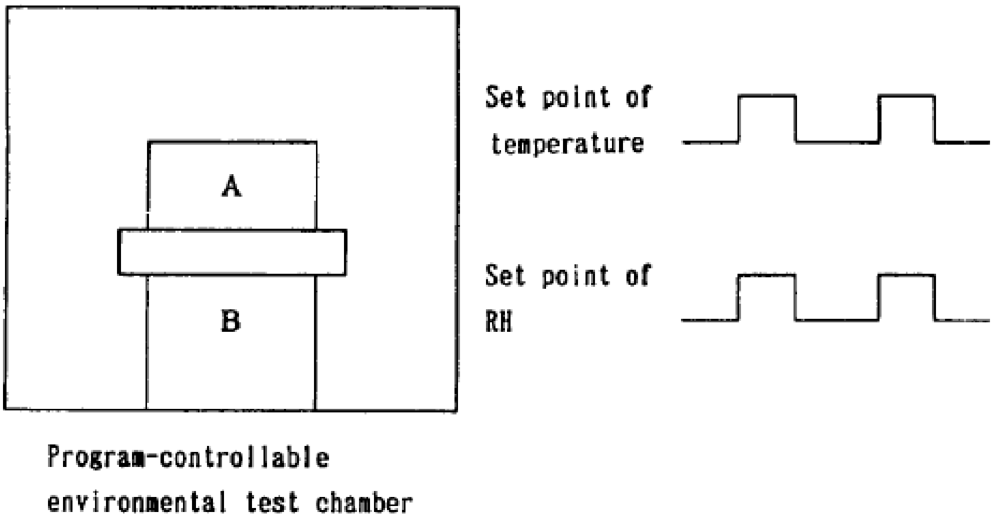

Fig.2 Schematic diagram of the apparatus used for cyclic enviromental test
A = anticipated stacking loads
B = corrugated containers filled with carton boxes for measurments

Table 1 Specifications of test samples

Model	Filling pattern of carton boxes	Spec, of carton boxes	Spec. of corrugated containers	Top clearance
I	Type 1	130×30×165mm Coated paper 310g/m^2	394×309×168mm KNN180/SCP180/KNN180g/m^2 A/F, Wrap around type with inner joint flap at top panel	0
I -var.	Type 1	130×30×165mm Coated paper 310g/m^2	394×309×168mm CN 170/SCP125/CN170g/m^2 A/F, Wrap around type with outer joint flap at side panel	0
II	Type 2	110×86×176mm Coated paper 450g/m^2	447×227×181mm KNN220/SCP180/ KNN220g/ m^2 A/F, Regular slotted type	6mm
II -var.	Type 2	110×86×176mm Coated paper 450g/m^2	447×227×190mm KNN220/SCP180/ KNN220g/m^2 A/F, Regular slotted type	15mm
III	Type 3	117×30×141mm Coated paper 310g/m^2	302×245×315mm KNN180/SCP160/KNN180g/m^2 A/F, Regular slotted type	12mm
III -var. 1	Type 4	117×30×141mm Coated paper 310g/m^2	302×245×315mm KNN180/SCP160/KNN180g/m^2 A/F, Regular slotted type	11mm adjusted by pads
III -var.2	Type 4	117×30×141mm Coated paper 310g/m^2	302×245×294mm CN170/SCP125/CN170g/m^2 A/F, Regular slotted type	11mm

ried out with program-controllable environmental test chamber (Fig. 2) that is controlled the temperature to ±0.3°C and the RH to ±2.5% for set point. Compression loads are applied through iron plates adjusted to the anticipated stacking loads on the commodity. That are 1177N to model I, 1187N to model II and 745N to model III.

Compression creep tests are made on two patterns of box arrangement. One is single corrugated container. The other is 2-tier model (Fig. 3) that

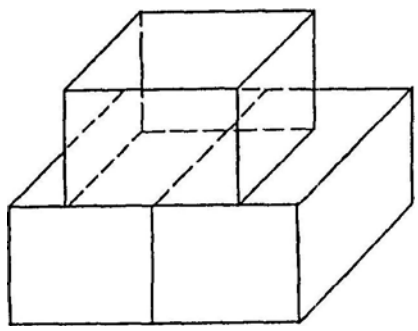

Fig.3 2-tier model for the evaluation of compression strength loss demonstrate the most simple form of compression strength loss during warehousing[3].

2.3 Cyclic environmental conditions

Two kinds of environmental condition" are set up to:

(a) 20°C, 65%RH(standard condition)

(b) 40°C, 90%RH(high temperature and high humidity condition).

The compression creep tests are carried out in the following 6 patterns of 24 hr period cyclic condition which starts and ends in standard condition.

(1) 4 hr cycle of 20°C, 65%RH and 40°C, 90%RH.

(2) 6 hr cycle of 20°C, 65%RH and 40°C, 90%RH.

(3) 8 hr cycle of 20°C, 65%RH and 40°C, 90%RH.

(4) 10 hr (20°C, 65%RH)-4 hr (40°C, 90%RH)-10 hr (20°C, 65%RH).

(5) ll hr (20°C, 65%RH)-2 hr (40°C, 90%RH)-ll hr(20°C, 65%RH).

(6) ll.5 hr (20°C, 65%RH)-1 hr (40°C, 90%RH)-ll.5 hr (20°C, 65%RH).

2.4 Evaluation

After 24 hr period test finished, the failure extent of carton boxes filled in corrugated containers is observed. The results of observation are ranked to 4 grades:

grade 1 = nothing of failure.

grade 2 = failure to a lesser extent.

grade 3 = failure to a medium extent.

grade 4 = failure to a greater extent.

3. Results and Discussion

The model I, II and III used in this study have been commercially sold commodities for more than

Table 2 Effect of various cyclic environmental condition tests for model I and I -var. on single corrugated container

Cyclic enviromental condition	Model	Total number of carton boxes	Distribution ratio of each failure grade			
			grade1	grade2	grade3	grade4
(1)	I I -var.	30 30				1.0 1.0
(2)	I I -var.	30 30	0.27 0.67		0.53 0.23	0.20 0.10
(3)	I I -var.	30 30	0.73 0.80		0.07	0.20 0.20
(4)	I I -var.	30 30		0.53 0.83	0.27 0.17	0.20
(5)	I I -var.	30 30	0.73 0.93	0.13 0.07	0.07	0.07
(6)	I	30	0.77	0.10	0.13	

Table 3 Effect of various cyclic environmental condition tests for model II and II -var on single corrugated container

Cyclic environmental condition	Model	Total number of carton boxes	Distribution ratio of each failure grade			
			gradel	grade2	grade3	grade4
(4)	II	10		0.20		0.80
(5)	II II -var.	10 10	0.20 0.70	 0.30		0.80
(6)	II II -var.	10 10	0.20 1.0		0.40	0.40

Table 4 Effect of cyclic environmental condition No.5 (5) test for model III , III-var.land III -var.2 on single corrugated container

Cyclic environmental condition	Model	Total number of carton boxes	Distribution ratio of each failure grade			
			gradel	grade2	grad e3	grade4
(5)	III III -var.1 III -var.2	40 40 40	0.90 1.0 0.97	0.05 0.03	0.05	

10 years. Failure does not have taken place for model I and III in the meantime, but has taken place for model II once for all in the high temperature and high RH environments of summer season.

3.1 Single corrugated container

The results of cyclic environmental tests for model I and I-var. (Table 2) reveal that:

(a) the compression creep strength of model I-var. is greater than that of model I in test condition No.2 (2)～No.5 (5).

(b) the test condition No.5 (5) and No.6 (6) provide the suitable time accelerated effect, corresponding to the stacking life of corrugated containers during warehousing.

The results of cyclic environmental tests for model II and II-var. (Table 3) reveal that:

(a) the compression creep strength of model II-var.1 is greater than that of model II in test condition No.5 (5) and No.6 (6).

(b) the test condition No.5 (5) and No.6 (6) provide the suitable time accelerated effect, corresponding to the stacking life of corrugated containers during warehousing.

The results of cyclic environmental tests for model III, III-var.l and III-var.2 (Table 4) reveal that:

(a) the compression creep strength of model III-var.-l and that of model III-var.2 are nearly

Table 5 Effect of cyclic environmental condition No,5(5) and No.6(6) tests for model I and I -var. on 2-tier model

Cyclic environmental condition	Model	Total number of carton boxes	Distribution ratio of each failure grade			
			grade 1	grad e2	grade3	grade4
(5)	I I -var.	90 90	0.91 1.0	0.09		
(6)	I	90	0.91	0.09		

Table 6 Effect of cyclic environmental condition No,5(5) and No.6(6) tests for model II and n -var. on 2-tier model

Cyclic environmental condition	Model	Total number of carton boxes	Distribution ratio of each failure grade			
			grade1	grade2	grade3	grade4
(5)	II II -var.	30 30	0.43 0.93	0.10	0.07 0.07	0.40
(6)	II II -var.	30 30	0.60 0.93	0.03 0.07	0.34	0.03

equal and greater than that of model III in test condition No.5 (5).

(b) the test condition No.5 (5) provides the suitable time accelerated effect, corresponding to the stacking life of corrugated containers during warehousing.

3.2 2-tier model

The results of cyclic environmental tests for model I and I-var. (Table 5) reveal that:

(a) the compressive creep strength loss of model I is greater than that of model I-var. in test condition No.5 (5).

(b) the test condition No.5 (5) and No.6 (6) provide the suitable time accelerated effect, corresponding to the stacking life of corrugated containers during warehousing.

The results of cyclic environmental tests for model II and II-var. (Table 6) reveal that:

(a) the compressive creep strength loss of model II is greater than that of model II-var. in test condition No.5 (5) and No.6 (6).

(b) the test condition No.5 (5) and No.6 (6) provide the suitable time accelerated effect, corresponding to the stacking life of corrugated containers during warehousing.

The results of cyclic environmental tests for model III and III-var.2 (Table 7) reveal that:

(a) the compressive creep strength loss of model III is greater than that of model III-var.2 in test condition No.5 (5).

(b) the test condition No.5 (5) provides the suitable

Table 7 Effect of cyclic environmental condition No,5(5) test for model III and III-var2. on 2-tier model

Cyclic environmental condition	Model	Total number of carton boxes	Distribution ratio of each failure grade			
			grade1	grade2	grade3	grade4
(5)	III	120	0.86	0.10	0.02	0.02
	III -var.2	120	0.99	0.01		

time accelerated effect, corresponding to the stacking life of corrugated containers during warehousing.

3.3 Criteria

There are some essential requisites as criteria for long term phenomena are performed.

Namely,

(a) the test methods for final evaluation should have suitable time accelerated effect.

(b) clear judgement should be achieved by these criteria.

The following steps are proposed to satisfy these requisites.

Step 1

Next values are multiplied to the distribution ratio of each 4 grade having been ranked according to the failure extent of carton boxes. The multiplier are 0 to grade 1,1 to grade 2, 3 to grade 3 and 5 to grade 4.

Step 2

The summation of weighted values to the failure extent of carton boxes are carried out.

$$w = x_2 + 3x_3 + 5x_4 \qquad (1)$$

Where

w = the sum of weighted values to the failure extent of carton boxes.

x_1 = the distribution ratio of carton boxes to the grade 1 failure.

x_2 = the distribution ratio of carton boxes to the grade 2 failure.

x_3 = the distribution ratio of carton boxes to the grade 3 failure.

x_4= the distribution ratio of carton boxes to the grade 4 failure.

$\Sigma x_i = 1.0$

Step 3

The composite summation of weighted values to the failure extent of carton boxes is defined as follows.

$$W = w_1 + w_2 \qquad (2)$$

where

W = the composite sum of weighted values to the failure extent of carton boxes for stacked corrugated containers.

w_1 = the sum of weighted values to the failure extent of carton boxes for the single corrugated containers test.

w_2 = the sum of weighted values to the failure extent of carton boxes for the 2-tier model test.

Step 4

The compression strength for corrugated con-

Table 8 Evaluation for the compression strength of filled corrugated containers by using the composite sum of weighted values to the failure extend of carton boxes on the cyclic environmental condition tests

Model	Cyclic environmental condition No.5 (5)			Cyclic environmental condition No.6 (6)		
	w_1	w_2	W	w_1	w_2	W
I	0.69	0.09	0.78	0.49	0.09	0.58
I –var.	0.07	0	0.07	0*	0*	0*
II	4.00	2.31	6.31	3.20	1.20	4.40
II –var.	0.30	0.21	0.51	0	0.07	0.07
III	0.20	0.26	0.46	-	-	-
III -var. 1	0*	0*	0*	0*	0*	0*
III -var. 2	0.03	0.01	0.04	0*	0*	0*

* Estimated value.
- Measurements are omitted.

tainers is determined by using guarantee index e as follows.

$$W \leqq \varepsilon \tag{3}$$

where

ε = the guarantee index for nothing of commodities failure during warehousing.

According to the above procedures, the results of calculations (Table 8) by using the data of Table 1~7 in the test condition No. 5 (5) and No. 6 (6) reveal that;

(a) the results of cyclic environmental test condition No. 5 (5) satisfy the essential requisites being necessary to criteria and coincide with the failure results of commercially sold commodities for more than 10 years.

(b) the suggested optimum values in the test condition No. 5 (5) are ε =1.0

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References

1) C. Murao, J. Pack. Sci. Technol., 4(4), 287(1995)
2) C. Murao, J. Pack. Sci. Technol., 4(4), 295(1995)
3) C. Murao, J. Pack. Sci. Technol., 5(1), 32(1996)
4) C. Murao, J. Pack. Sci. Technol., 5(1), 43(1996)
5) C.MuraoJ.Pack.Sci.Technol.,5(2) , 99(1996)
6) V. L Byrd, Tappi, 55(2), 247(1972)
7) V. L. Byrd, Tappi, 55(ll), 1612(1972)
8) V. L. Byrd and J. W. Koning, Jr., Tappi, 61(6), 35 (1978)
9) V. L. Byrd, Tappi, 67(7), 86(1984)

10) V. L. Byrd, Tappi, 69(10), 98(1986)
11) C.H. Leake.Tappi, 71 (10), 71(1988)
12) A. Boonyasarn, B. R. Harte, D. Twede and J. L. Lee, Tappi, 75(10), 79(1992)
13) U. I. Ievans, "Performance and Evaluation of Shipping Containers" (Edited by G. G. Maltenfort), Jelmar Publishing Co., Inc., pl56(1989)

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食品外装段ボール箱圧縮強さに及ぼす温湿度環境の影響：モデル化と評価法

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カートン個箱入り外装段ボール箱圧縮強さに及ぼす、サイクリック温湿度環境の影響を検討した。一定温湿度環境で測定したカートン個箱入り外装段ボール箱圧縮強さ特性を、各種サイクリック環境条件化で確認した。これらの確認実験の結果、現実の段ボール箱の積付寿命に対応する、適度の時間促進効果を有するサイクリック環境モデル条件が存在することが判明した。このようにモデル化したサイクリック環境条件で圧縮強さを確認すれば、一定環境条件化のテストにより設計された外装段ボール箱圧縮強さの最適設定の妥当性を評価できることが判明した。

キーワード：外装段ボール箱、カートン個箱、圧縮強さ、一定環境条件、サイクリック環境条件、積付寿命、最適条件

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Method of Optimum Design for Compression Strength of Corrugated Containers for Food Products

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The structure of the mathematical model for the compression strength of corrugated containers with contents is investigated. The characteristics of 3-dimensional curved surface which consists of 3 state variables in the model are revealed. The top clearance and the board composition of corrugated containers filled with carton boxes are two controllable factors. The graphical method of optimum design for those is demonstrated. The flow chart of optimum design for the compression strength of corrugated containers with contents is proposed by using this method and the cyclic environmental model condition. It is indicated that the theme of this study is similar to the bimatrix game of game theory.

Keywords: Corrugated container, Carton box, Compression strength, Mathematical model, Optimum design, Graphical method, Top clearance, Board composition

1. Introduction

The mathematical model was proposed for the compression strength of corrugated containers with contents in the previous paper[1]. Furthermore, the characteristics of the model was described in detail for the various carton boxes of contents[2),3)].

Top clearance means the clearance between the top panel of corrugated containers and contents. The correlative relationship between the top clearance and the coefficients of the model was explained[4),5)].

Moreover, the cyclic environmental model conditions for the real stacking life were studied to evaluate which was the optimum design under the constant environmental condition[6)].

In this paper, the structure of the model is analyzed in the first half. The graphical method of optimum design for the compression strength of corrugated containers filled with carton boxes are shown by using this analyzed results.

Finally, the flow chart of optimum design is proposed, which is capable of adapting not only to carton boxes but also to various contents.

2. Structure of Mathematical Model

The mathematical model which was studied in

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the previous papers[1)-5)] was shown as follows.

$$C = B + \alpha D \qquad (1)$$

$$C = \beta A \qquad (2)$$

where

C = the compression strength of filled corrugated containers.

B = the compression strength of contents which are filled in corrugated containers.

D = the compression strength of empty corrugated containers.

A = the anticipated stacking loads to the bottom corrugated containers in storage.

α = coefficient which means the contribution of empty corrugated containers to the compression strength of filled corrugated containers.

β = coefficient which means compensating factor for the compression strength loss during warehousing being defined by 2-tier model[3),5)].

The dimension and the board composition of carton boxes will be determined at the commodities designing.

Therefore, B is a given factor in the equation (1).

A, which is determined by distribution condition, is also given factor similarly in the equation (2).

The length and width of corrugated containers are given by the filling pattern of carton boxes.

Two factors still remain to design corrugated containers. Namely, one is the height which is equivalent to the top clearance of corrugated containers and another is the composition of the corrugated board.

The type of corrugated containers is restricted only the regular slotted one in this paper. Wrap-around type is only used in the case of the compression strength of contents is sufficient to the stacking loads. Therefore, the compression strength design of corrugated containers is omitted.

The above mentioned two factors to design are expressed as follows.

δ_0 = the initial top clearance of filled corrugated containers without stacking loads.

δ_{0i} = the i-th value of δ_0 which is constituted of n cases of various top clearances.

M = the board composition of corrugated containers.

M_j = the j-th case of M which is consisted of m cases of various composition.

The equations of the model, corresponding to the combination of δ_{0i} and M_j, are shown as follows.

$$C_{ij} = B + \alpha_{ij} D_j \qquad (3)$$

$$C_{ij} = \beta_{ij} A \qquad (4)$$

where

C_{ij} = C which corresponds to the combination of δ_{0i} and M_j.

D_j = D which corresponds to M_j.

α_{ij} = α which corresponds to the combination of δ_{0i} and M_{ij}.

β_{ij} = β which corresponds to the combination of δ_{0i} and M_j.

It is supposed that D_j is not affected by δ_{0i} which varies within a practical limit. Moreover, we should notice that δ_{0i} has a perfect degree of freedom but M_j has an allowable lower limit to support the stacking loads.

The theme of this study is described that it is to design the minimum D_j by the optimum combination of α_{ij} and β_{ij}. The combination of α_{ij} and β_{ij} is corresponding to δ_{0i} and M_j. Therefore, the theme is eqivalent to design the minimum D_j by the optimum combination of δ_{0i} and M_j.

The equation (3) and (4) give

$$D_j = A(\beta_{ij} - B/A)/\alpha_{ij} \tag{5}$$

The standardization of the equation (5) gives

$${}^sD_j = D_j/0.5A = 2(\beta_{ij} - B/A)/\alpha_{ij} \tag{6}$$

where

${}^sD_j$ = the state variable derived from the standardization of the equation (5) by D_j = 0.5A, which corresponds to D_j at B/A=0.5, α_{ij}=1.0 and β_{ij}=l.0.

The equation (5) and (6) provide the 3-dimensional curved surface to the model. Equation (6) gives the followings.

If β_{ij}=constant, then ${}^sD_j$ is proportional to $1/\alpha_{ij}$.

If α_{ij}=constant, then ${}^sD_j$ is linear to β_{ij}.

Similarly, equation (6) gives the sensitivity of ${}^sD_j$ corresponding to variation of α_{ij} and β_{ij} as follows.

If β_{ij} = constant, then

$$\frac{d\,{}^sD_j}{d\alpha_{ij}} = -2(\beta_{ij} - B/A)\ /\ \alpha_{ij}^2.$$

If α_{ij} = constant, then

$$\frac{d\,{}^sD_j}{d\beta_{ij}} = 2/\alpha_{ij}.$$

It is indicated that the contours of ${}^sD_j$, which are projected to β_{ij}-α_{ij} plane, show the following characteristics.

[Corollary 6-1] The contours of various ${}^sD_j$ at constant B/A are straight lines and intersect at a point of B/A of β_{ij}-axis.

[Corollary 6-2] The contours of same ${}^sD_j$ at various B/A are parallel straight lines.

[Corollary 6-3] The slopes of straight lines are $2/{}^sD_j$, equal to A/D_j.

where

[Corollary 6-1] means Corollary-1 of the equation (6).

If δ_{0i} and Mj are not optimum design, the following equations are presented instead of the equation(4).

$$C_{ij} = \beta_{ij} KA \tag{7}$$

where

K = the deviation factor which expresses the degree of deviation from the optimum design.

It is shown by the follows.

If C_{ij} > optimum C_{ij}, then K>1.

If C_{ij} > optimum C_{ij}, then O<K<1.

Comparing the equation (7) to (4), we indicate the following characteristics.

[Corollary 7] KA in the equation (7) is equivalent to A in the equation (4).

The equation (7) shows that the imaginary stacking loads of the non-optimum design are K times bigger as compared with the real stacking loads of the optimum design.

The equation (3) and (7) give

$$D_j = KA\ (\beta_{ij} - B/KA)/\alpha_{ij} \tag{8}$$

The standardization of the equation (8) gives

$$^{s}D_j = D_j/0.5KA = 2(\beta_{ij}-B/KA)/\alpha_{ij}. \quad (9)$$

where

$^{s}D_j$ = the state variable derived from the standardization of the equation (8) by D_j = 0.5KA, which corresponds to D_j at B/KA=0.5, α_{ij}=1.0, and β_{ij} = 1.0.

Replacing A of [corollary 6-1, 6-2, 6-3] by KA, we have a generalized form of corollary.

3. Graphical Method of Optimum Design

The graphical method of optimum design is investigated for the corrugated containers filled with carton boxes.

3.1 Graph of mathematical model

The studies on the characteristics of α_{ij} and β_{ij} for the corrugated containers filled with carton boxes were made under the constant environmental condition of 20°C, 65%RH/40°C, 90%RH[2)-5)]. Those gave the following results.

1) The range of α_{ij} is $0 \leqq \alpha_{ij} \leqq 1.2$.
2) The function form of α_{ij} is the monotone increasing function of δ_{0i} within the range of $0 \leqq \alpha_{ij} \leqq 1.0$.
3) There is a large probability that the upper limit of α_{ij} increases to 1.2.
4) The range of β_{ij} is $1.0 \leqq \beta_{ij} < 4.0$.
5) The function form of β_{ij} could not be described in unitary one.

The following two forms of β_{ij} were suggested.

① β_{ij} is the monotone increasing function of β_{0i}.

② β_{ij} has a minimum within the practical range of β_{0i}.

Considering those results, the variables are set as the range of α_{ij} and β_{ij} to α_{ij}= 0.6-1.2 and β_{ij}= 1.0 ~4.0 in this study. β_{ij}= 4.0 corresponds to the case of empty corrugated containers of which the width/length ratio of dimension is small, for example 0.5.

Fig. 1 shows the 3-dimensional curved surface of the equation (6) of which B/A = 0.50.

Fig. 2 shows the case of B/A = 0.25 and 0.75. Fig.1 and 2 reveal the followings.

1) $^{s}D_j$ is maximum at α_{ij}= 0.6 and β_{ij} = 4.0 and minimum at α_{ij}= 1.2 and β_{ij}= 1.0. Those are monotone increasing curved surfaces from a point of minimum $^{s}D_j$ to a point of maximum $^{s}D_j$.
2) If β_{ij} is small, then $^{s}D_j$ is also small. If β_{ij} increases, then $^{s}D_j$ increases too. In the1 same β_{ij} section, if α_{ij} decreases, then $^{s}D_j$ increases rapidly.
3) If B/A increases, then the curved surface moves to the positive direction of β_{ij}-axis by the increment of B/A.

The matters to take care are summarized as follows.

1) Small β_{ij} is the first.
2) Large α_{ij} is the second. Considering the function form of β_{ij} to δ_{0i}, β_{ij} has the various level of lower limit in accordance with the packaging style. Therefore, if it is difficult to keep β_{ij} small, then large α_{ij} is important.
3) If the increment of B/A is 0.5, then the decrement of $^{s}D_j$ is approximately 1.0~2.0.

Fig. 3 shows the contour map of $^{s}D_j$, which is projected to β_{ij}-α_{ij} plane, in the case of B/A=0.50.

If B/A=0.75, then the contour lines of $^{s}D_j$, which is shown in Fig. 3, moves to the positive direction of the β_{ij}-axis by 0.25. If B/A = 0.25, then the contour lines of $^{s}D_j$ moves to the negative direction of the β_{ij}-

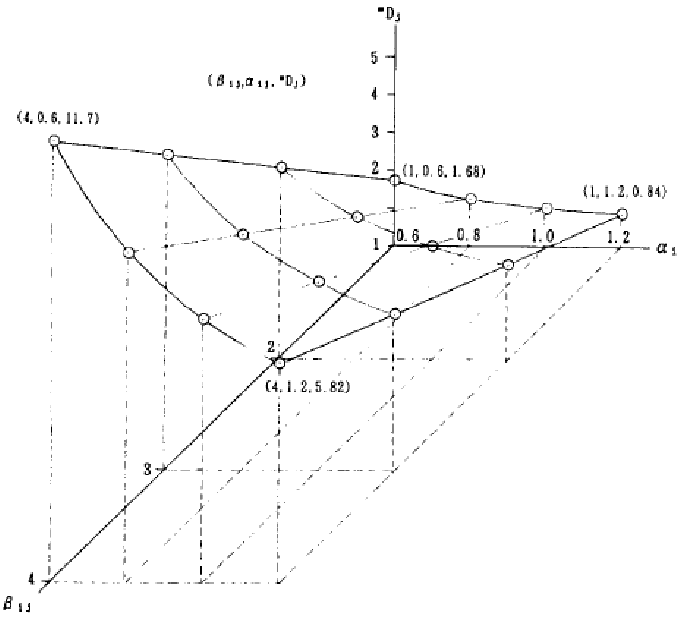

Fig.1 3-dimensional curved surface of the model (B/A = 0.50)

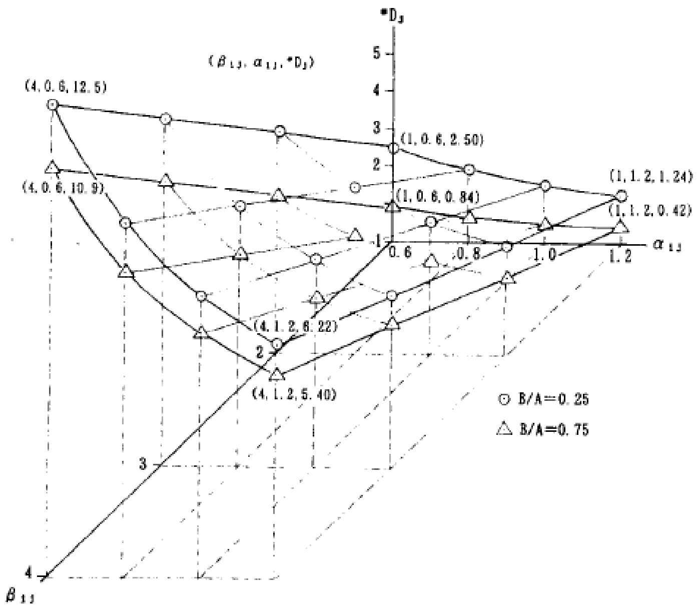

Fig.2 3-dimensional curved surface of the model (B/A=0.25, 0.75)

axis by 0.25.

3.2 Graphical method of optimum design for δ_0

The graphical method of optimum design for δ_0 which is the initial top clearance of corrugated containers filled with carton boxes is proposed.

The studies was made on the characteristics of coefficients a and (3 in the equation (1) and (2)[2)-5)]. The data of those are applied to explain the procedures on the graphical method of optimum design.

Fig. 4 shows the filling patterns of the carton boxes in corrugated containers. Table 1 shows the specifications of test samples.

3.2.1 Determination of environmental conditions and creep time for the measurement of B

The contour map of $^{S}D_j$ is used to the graphical method. A is given. Thus we should determine by what conditions B is measured. The condition for measurement of B is determined to 5 min. creep time in 40°C, 90%RH by the following study.

(1) Case I

A=1187N, B=412N, C=1177N

then C/A=0.99, B/A=0.35

where

A = the anticipated stacking loads to the bottom corrugated containers in 3 high stacks of palletizing.

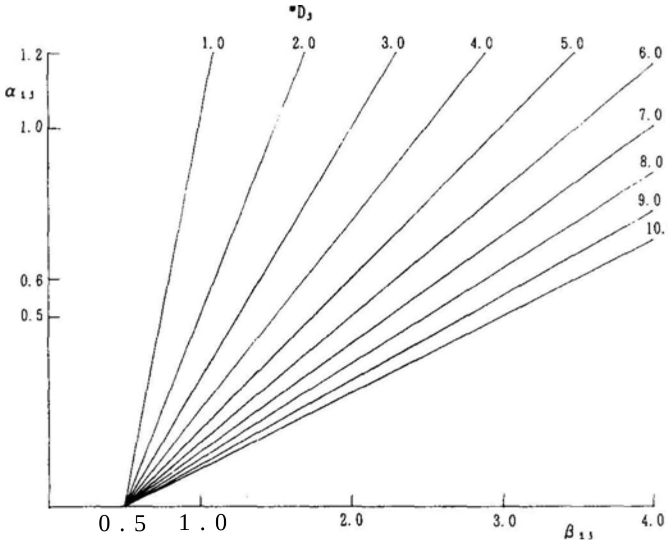

Fig.3 Contour map of sDi protected to pij-cxij plane (B/A = 0.5)

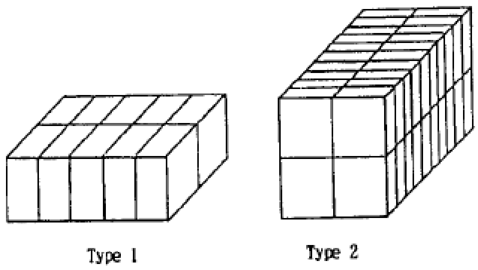

Fig.4 Filling pattern of the carton boxes in corrugated containers

B = the compression strength of carton boxes filled in corrugated containers at 40°C, 90%RH and 5 min. creep time conditions.

C = the compression strength of corrugated containers filled with carton boxes at the same conditions of B.

According to the survey of the failure of the corrugated containers which is classified as Case I during distribution, failure take place once for all in the high temperature and the high relative humidity environments.

By considering C/A=1.0 as the approximate index for optimum design, then the above C/A=0.99 reflects accurately the results of a long term distribution.

(2) Case II

A=745N, B=294N, C=843N

then C/A=1. 13, B/A=0.39

The commodities classified as Case II also have been sold for more than 10 years. Failure does not have taken place in the term. Considering C/A=1.0 is the approximate index for optimum design, then the board composition of the corrugated containers classified as Case II is over specifications to some extent for compression strength. Therefore, the above C/A=1.13 also reflects accurately the results of a long term distribution.

3.2.2 Optimum design procedure for δ_{o}

The procedure in optimum design for $\delta_{()}$ are shown as follows.

Step l

Plot the set of the measured value of α_{ij} and β_{ij} corresponding to the variation of $\delta_{()i}$ to the β_{ij}-α_{ij} plane, and draw the locus curve.

Step 2

Give the tangent to the locus curve from a point of B/A on the β_{ij}-axis.

The touch point of the tangent to the locus curve gives the suggested optimum $\delta_{()}$ ($\delta_{(),opt}$).

Step 3

Table 1 Specifications of test samples

Case	Filling pattern of carton boxes	Spec. of carton boxes	Spec, of corrugated containers
I	Type l	110×86×176mm Coated paper 450g/m^2	447×227×190mm KNN220/SCP180/KNN220g/m^2 A/F Regular slotted type
II	Type 2	117×30×141mm Coated paper 310g/m^2	302X245X315mm KNN180/SCP160/KNN180g/m^2 A/F Regular slotted type

The slope of the tangent gives the suggested minimum $^{s}D_j$ (D_{opt}).

Fig. 5 shows the graphical method of finding δ_0,opt for the Case I. The points of δ_{01}, δ_{02}, δ_{03}, δ_{04} and δ_{05} indicate the set of measured α_{ij} and β_{ij} corresponding to the variation of δ_0 which are 6, 9, 12, 15 and 20mm. B/A is expressed as 0.35.

The $\delta_{O,opt}$ is approximately 14mm and the D_{opt} is 2.3.

Fig. 6 shows the same procedure to the Case II. A vriation of δ_0 are 5, 8, ll, 14 and 20mm. From Fig. 6, it can be stated that if B/A is expressed by 0.39, then the $\delta_{0,opt}$ is approximately 13mm and the D_{opt} is 1.2. B/A = 0.39 for the Case II corresponds to C/A=1.13 in accordance with the previous section study. If β_{ij}=1.0 is the index for optimum design, then K=1.13 is derived from the equation (7). Replacing A by KA in accordance with the corollary (7), B/KA=0.35 can be obtained instead of B/A=0.39.

Therefore, it is correct that the tangent for β_{ij}=1.0 intersects a point of B/KA=0.35 on the β_{ij}-axis.

Fig. 7 shows how variation of K exerts an influence on $_{0,opt}$. If K is infinity, equal to B/KA=0, then

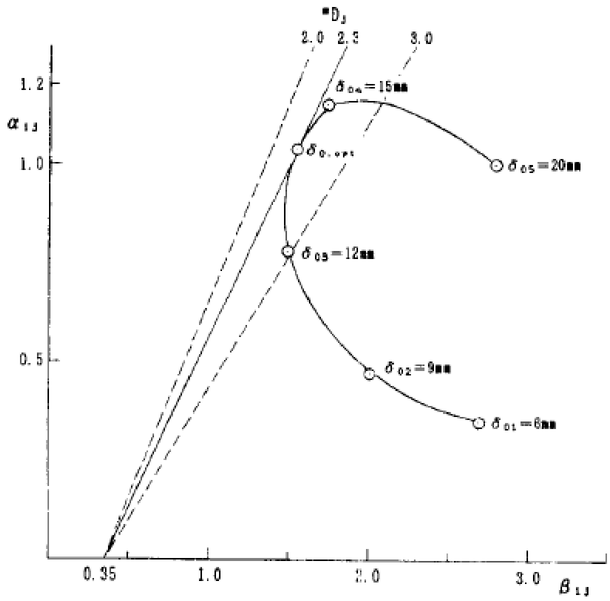

Fig.5 Locus curve of δ_0 and procedure in getting $\delta_{0,opt}$ for Case 1 (B/A = 0.35)

the $\delta_{0,opt}$ is approximately 13mm. If K is 0.5, equal to B/KA=0.78, then the $\delta_{0,opt}$ is approximately 12mm.

It is revealed that a large variation of K or the measurement error of B exerts an influence to a lesser extent on $\delta_{0,opt}$ for the Case II. It is suggested that there is no significant difference for the Case I by the observation of the locus curve in Fig. 5.

Finally, the fixed value of B/A, for example 0.5, is

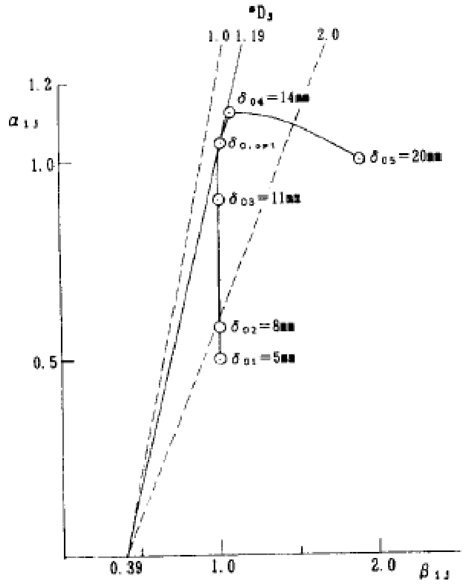

Fig.6 Locus curve of δ_{0i} and procedure in getting $\delta_{0,opt}$ for Case II (B/A=0.39)

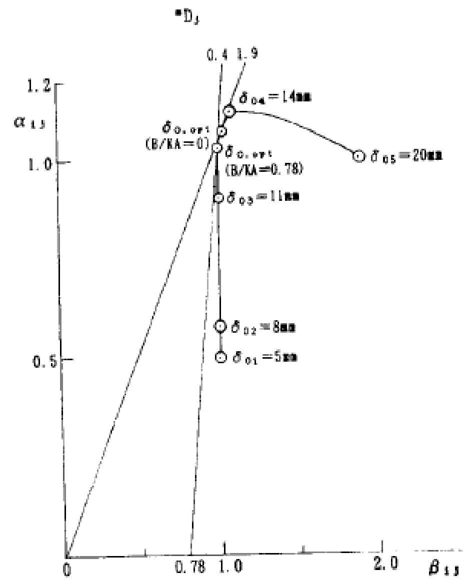

Fig.7 Locus curve of δ_{0i} and an influence of variation of K on $\delta_{0,opt}$ for Case II (B/KA=0, 0.78)

capable of using for finding out the approximate value of δ_0, opt in the Case I and II.

How does the variation of K or the measurement error of B exert an influence on D_{opt}?

The intersection point of contours at the β_{ij}-axis moves from B/A to B/KA in accordance with the [corollary 7,6-1]. The slope of contours is multiplied by K in accordance with the [corollary 7, 6-3].

Consequently, the slope of the tangent touching the locus curve, equivalent to D_{opt}, is shown as follows.

If K>1, then Dopt is over K times bigger as compared with that of the optimum design at K=1.0.

If O<K<1, then D_{opt} is K times smaller as compared with that of the optimum design.

Thus the variation of K or the measurement error of B exerts an significant influence on D_{opt}.

3.3 Optimum design procedure for M

The optimum design for M can be obtained by using the cyclic environmental model condition for compression strength of corrugated containers in the previous paper[6) .

The procedure in the optimum design of M are shown as follows.

Step l

Determine the appropriate M and find out the approximate $\delta_{(),opt}$.

Step 2

Test the combination of the M and the approximate $\delta_{0,opt}$ by the cyclic environmental model condition[6)] .

If W>1.0, then the higher compression strength of M .

If W<0.1, then the lower compression strength of M.

where

W= the composite sum of weighted values to the failure extent of carton boxes for stacked corrugated containers[6)] .

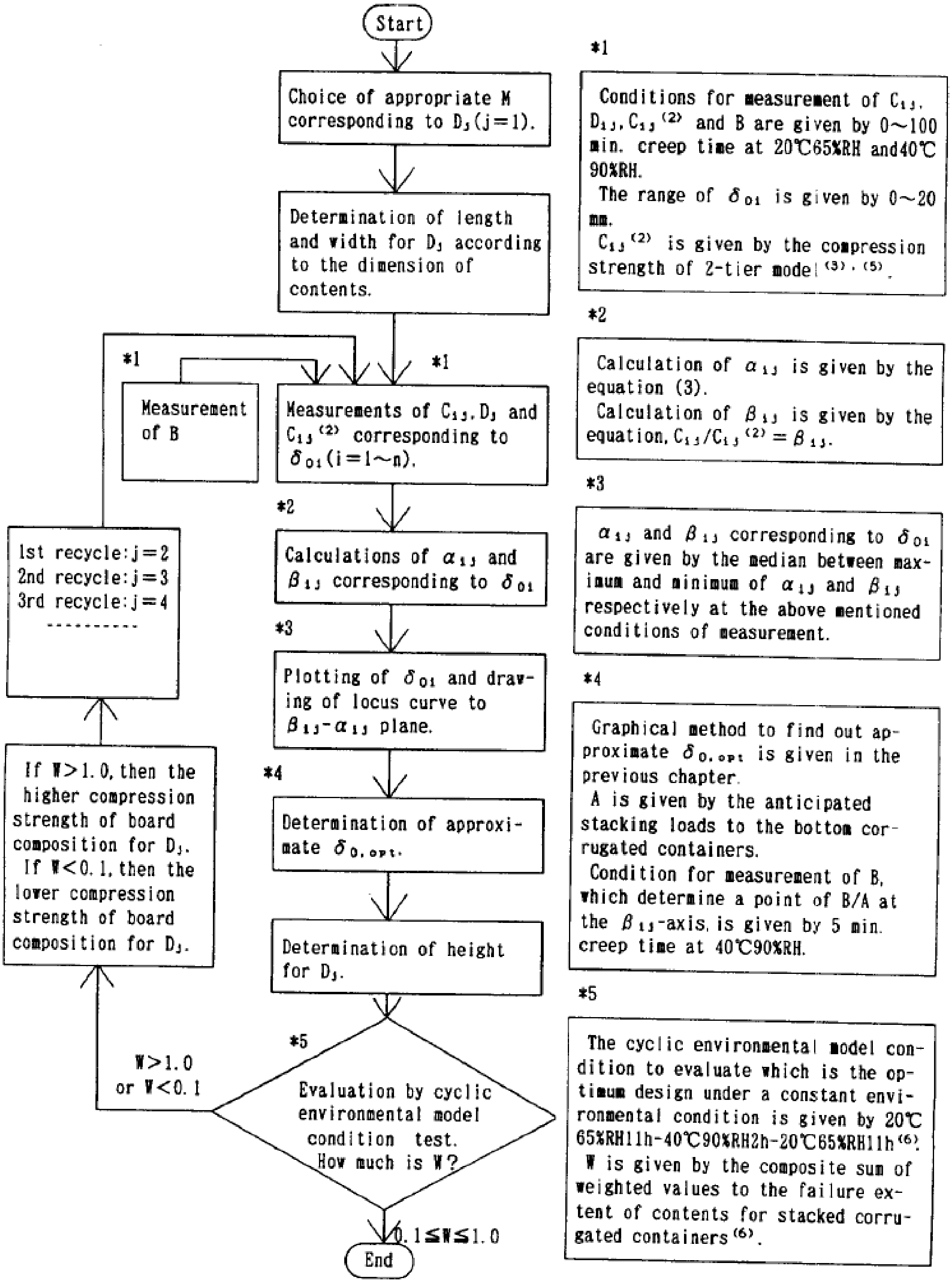

Fig.8 Flow chart of optimum design for the compression strength of corrugated containers with contents

Step 3

Back to step 1 untill W reaches to the range of $0.1 \leqq w \leqq 1.0$

It is interpreted that W>1 corresponds to O<K<1 and W<0.1 corresponds to K>1 in this paper.

The value of K depends on the distribution process.

Therefore, the above mentioned relationship of W to K varies similarly according to the distribution process. Deviation of ${}^{S}D_j$ from D_{opt} is attributed to the error of K, B or initial deviation of M from the optimum board composition. This can be corrected by the procedure in step 2. Finally, $\delta_{(),opt}$ and D_{opt} are obtained by the recycle of the step 1 and 2.

4. Conclusion

Fig. 8 shows the flow chart of optimum design for the compression strength of corrugated containers with contents. The equations and corollaries are established by not only carton boxes but also various contents. It is suggested that the methodology of this study is capable of being adapted to the corrugated containers with various contents.

The function forms of various locus curve of $\delta_{\{\}i}$ shall be ensured by further data accumulation. It means that α_{ij} and β_{ij} for the various styles of packaging and various board compositions shall be measured.

As the results of the data accumulation, a number of trial cycle to reach the optimum design shall be decreased, for example 1 to 3 cycles. From the graphical method of this study, there is only one equilibrium point foe the Case I and II. But it does not be concluded that there is only one equilibrium point for all cases of corrugated containers with contents. There is a possibility to have two or three equilibrium points. It can be expected that the function forms of the locus curve of $\delta_{()i}$ more clearly revealed by the accumulation of data.

The theme of this study is similar to the problem of game theory which is a bimatrix game of α_{ij} and β_{ij}[7]. In the case of multi-equilibrium points, the strategy of game theory is to be reffered.

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References

1) C. Murao, J. Packaging Sci. & Technol. Japan, 4(4), 287 (1995)
2) C. Murao, J. Packaging Sci. & Technol. Japan, 4(4), 295 (1995)
3) C. Murao, J. Packaging Sci. & Technol. Japan, 5(1), 32 (1996)
4) C. Murao, J. Packaging Sci. & Technol. Japan, 5(1), 43 (1996)
5) C. Murao, J. Packaging Sci. & Technol. Japan, 5(2), 99 (1996)
6) C. Murao, J. Packaging Sci. & Technol. Japan 5(3), 191 (1996)
7) T. Nishida, "Game theory" JUSE Press, pl40, (1992)

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