# A Logistics Model for Post-Consumer Waste Recycling

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In this paper, we propose a logistics model for post-consumer packaging waste recycling (Grid City Model). This model can predict the transport distance and the number of trucks needed to collect post-consumer packaging waste for recycling. The parameters needed are population, area, waste discard unit, number of collection stations, size of a collection truck, frequency of collection. Trade-off relation between the service level of waste collection and environmental load or the number of trucks is explicitly described in the Grid City Model.

Both the unit transport distance (D/W) and the number of collection trucks per a unit waste (M/W) are described as a sum of the "density effect" and the "scale effect": the "density effect depends on the effective capacity of the collection truck (q) and independent of the waste discard unit (u) nor the collection, frequency (f); the "scale effect" depends on both u and f and independent of q.

Both D/W and M/W of sorted collection of PET bottle is much larger than those of the household waste in a typical case.

Keywords: Recycling, Sorted collection, Logistics, Transportation, Model, Post-consumer waste,

#### Introduction

This paper describes the essence of our model of logistics for post consumer waste packaging (Grid City Model). The model was developed by the author for a joint project sponsored by Japan Consumers' Co-operative Union and Environmental Agency (Anonymous, 1995), and conducted by the Nomura Research Institute Ltd. (NRI). The objectives of the study were to examine quantitatively the effect of characteristics of a region; e.g. population density, waste discard unit, on the environmental load of recycling, to compare the inventory of different recycling system; material recycling, thermal recycling, and chemical recycling.

The recycling was a key problem from the early stage of LCA study (e.g. MRI, 1974; MRI, 1975). Among LCA studies of recycling of postconsumer waste (hereafter referred to as PCW), transportation process has been always an irksome process: because the environmental load of transportation in the recycling is suspected to be large and the transportation process in a recycling

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of PCW was usually described by a simple scenario such as an assumption of the transportation distance; this means that results directly depend on the assumption.

In the present paper, the author attempts to give a clear and transparent structure to the collection and transportation process for a LCA study.

The present paper can be divided into two parts. In the first part, we explain the key concepts of the model, and show the explicit expression. In the second part, we show how we determined the parameters based on data from 30 municipalities in Japan and compare the observed data and the model predictions.

#### Model

A schematic diagram of the Grid City Model is depicted in Figure 1. In this figure, each open block are parameters or various, and arrows indi-



Fig. 1 A schematic diagram of "Grid City Model"

cate causal relationships. In the largest gray block, we collected the output variables; D, M, D/W, M/W. The parameters which characterize the city are in the top of the right column; A, £, P,W. This group of parameters should be given for a certain city, however, an average value of E, for 30 municipalities in Japan is given in this paper and the average value can be substituted to it. The parameter in the top of the left column is p,: the apparent density of the i-th waste in the collection trucks which was calculated from data on composition of household waste (Kyoto-shi Seisoukyoku, 1994). In the bottom of the left column, policy variables are shown; q, N, f. We consider these variables are policy variables for improvement. We grouped the parameters which relate to the collection productivity in the bottom of the right column;  $v_{P}$ ,  $t_{L}$ , k.

It may be interesting to trace the arrows start

from parameters of collection productivity, one will find that there is no causal path between transport distance and these parameters. Although everyone will agree that collection productivity should be a major factor in the economy of collection system, this means that the environmental load of transport is independent from the collection productivity. This can be understood if we see the waste material flow from the Lagrangean point of view; when we observe the transport distance riding on a certain waste discarded at a certain point, the distance should depend on only state variables such as geometric parameters.

Let us consider a city which has a regular grid type configuration (Figure 2). In this city, a collection truck starts from the recycling center which locates in the center of the square area drives to the collection area. In the collection area, the truck collects the waste at collection stations until its



Fig.2 The geometry of collection stations, collection center, and grid

maximum capacity: it means that a collection area is dependent on the capacity of the truck. After filling its load space, the truck returns to the recycling center. The collection truck repeats this collection trip several times a day.

The distance of a collection trip is expressed as:

$$\boldsymbol{d}^{G} = \boldsymbol{d}_{1}^{G} + \boldsymbol{d}_{2}^{G} \tag{1}$$

In equation (1), superscript G denotes grid city model, subscripts 1 and 2 denote driving distance in the collection area, and the distance between recycling center and the collection area, respectively. The average value of  $d^{\circ}$  can be obtained as:

$$\overline{d}_{2}^{C} = \frac{4 \cdot \left\{ \sum_{i=1}^{J} \sum_{j=1}^{J} \left( i - \frac{1}{2} + j - \frac{1}{2} \right) \right\} \cdot L_{1}}{M} \cdot 2$$
(2)

Similarly, the average value of  $d_1^G$  is,

$$\overline{d_1^G} = m \sqrt{\frac{A}{N}} \tag{3}$$

In equation (3), m denotes the number of collection stations which can be served in a trip, A denotes area per a collection center, N denotes the number of collection stations per a collection center. When we consider a macro balance of waste discarded and the waste collected a year, m can be expressed by q, N, f, and Was:

$$m = \frac{f \cdot q \cdot N}{W} \tag{(4)}$$

Since A is equal to  $L^2_{\ c}$  , the average transport distance of a trip is described as:

$$\overline{d}^{G} = \left(1 + \frac{f \cdot q \cdot \sqrt{N}}{W}\right) \cdot \sqrt{A} \tag{5}$$

In the above equation,/denotes frequency of collection service.

It is evident that the actual distance will differ from this value because the actual road configuration is not a regular grid. We introduce a factor which correct this effect ( $\xi$ ,: tortuosity factor).

$$\xi = \frac{D^a}{D^G} \tag{6}$$

In equation (6), D denotes total driving distance of collection trucks of a recycling center per year, superscript o denotes the observed value. Since the total number of the collection trips a year is given by W/q, tortuosity factor of the i-th city is given as:

$$\xi_i = \frac{D_i^o}{\left(\frac{W_i}{q_i} + f_i \cdot \sqrt{N_i}\right) \cdot \sqrt{A_i}}$$
(7)

Therefore, if we have an average value of the tortuosity factor, we can estimate the total transport distance D from the equation below.

$$D = \xi \cdot D^G = \xi \cdot \left(\frac{W}{q} + f \cdot \sqrt{N}\right) \cdot \sqrt{A} \tag{8}$$

Let us introduce population (P) instead of W using the waste discard unit (u).

$$W = u \cdot P \tag{9}$$

When we substitute equation (9) into (8) and divide it by W, we get:

$$\frac{D}{W} = \xi \cdot \left( \frac{\sqrt{A}}{q} + \frac{f}{u} \cdot \sqrt{\frac{N}{P}} \cdot \sqrt{\frac{A}{P}} \right)$$
(10)

From this equation, the transport distance per a unit mass of waste is a decreasing function of population density (P/A), when the ratio of the number of collection stations and population (N/P) is kept constant. This equation can be interpreted as showing a quantitative trade-off relationship between the environmental load and the service level of waste collection: when f or N increase, the service level will increase but the environmental load will increase.

Although the number of trucks (M) is independent of environmental load, it is important for the economy of the system. Let us consider what determines M.

The time necessary for a collection trip  $(t_{-})$  is defined as the sum of the driving time, loss time $(t_{-}^{L})$ , and the loading time. Assuming that the velocity is constant  $(v_{-}^{P})$ , a certain amount of loss time is necessary per a collection station, and the loading time is proportional to the amount of the waste, we have an expression of  $t_{-}^{T}$  as below

$$t^{T} = \frac{\overline{d^{C}} \cdot \xi}{v_{p}} + \frac{q \cdot N \cdot f}{W} \cdot t_{L} + \frac{q}{k}$$
(11)

From mass balance, the average number of collection trips of a collection truck per a operation day (n) is given as:

$$n = \frac{W}{M \cdot F \cdot q} \tag{12}$$

Since the product of n and  $t^{T}_{.}$ , M can be given as below from rearrangement of the product.

$$M = \frac{1}{F \cdot t^{D}} \left\{ \frac{W \cdot \xi}{q \cdot v_{p}} \left( 1 + \frac{f \cdot q}{W} \cdot \sqrt{N} \right) \cdot \sqrt{A} + f \cdot N \cdot t_{L} + \frac{W}{k} \right\}$$
(13)

Dividing equation (13) by (9) followed by substitution of (9) yields:

$$\frac{M}{W} = \frac{1}{F \cdot t^{D}} \cdot \left\{ \frac{\xi \cdot \sqrt{A}}{v_{p} \cdot q} + \frac{\xi \cdot f}{v_{p} \cdot u} \cdot \sqrt{\frac{N}{P}} \cdot \sqrt{\frac{A}{P}} + \frac{f \cdot t_{L}N}{u P} + \frac{1}{k} \right\}$$
(14)

This equation shows that the number of collection trucks per a unit mass of waste is a decreasing function of population density when N/P is constant. As is expected, the increase in consumer's convenience: increase in f or N, results in increase in M/W which can be interpreted as a substitute variable of unit collection  $c \ o \ s \ t$ .

### **Identification of parameters**

Model parameters:  $\xi$ ,  $v_P$ ,  $t_L$ , and k are determined by the data of 30 municipalities in Japan. The population of the cities vary from thirty thousand up to three million: 3 cities over one million, 6 cities over five hundred thousand, 12 cities over two hundred thousand, 7 cities over fifty thousand, and remaining two cities are between twenty thousand and fifty thousand.

The tortuosity factor of a certain city was calculated from equation (6), and a simple average of the tortuosity factor was calculated.

The parameters which represent collection productivity:  $v_p$ ,  $t_L$ , and k are determined by a multiple regression analysis. In order to reflect the actual condition, the model was slightly modified for analysis. Since the number of trips per a working day (n) should be an integer, n' is defined as:

$$n_{i}' = Ceiling\left(\frac{W_{i}}{M_{i} \cdot F_{i} \cdot q_{i}}\right)$$
(15)

where Ceiling(x) denotes the minimum integer which is not smaller than x. The number of stations collected in a collection trip (m) is expressed

by n' as:  

$$m_i' = \frac{N_i f_i}{n_i' \cdot F_i \cdot M_i}$$
(16)

The actual driving distance of a trip  $(d^0_{-})$  can be calculated from the observed data as

below.

$$d_i^o = \frac{D_i^o}{M_i F_i n_i^*} = \xi \cdot d_i^G \tag{17}$$

We define the effective capacity of the collection truck as:

$$q_i' = \frac{W_i}{M_i F_i n_i'} \tag{18}$$

Substitution of equations (16), (17), and (18) into (11) and rearrangement yields:

$$t_i^{\mathsf{T}} = \boldsymbol{\alpha} \cdot \boldsymbol{x}_i + \boldsymbol{\beta} \cdot \boldsymbol{y}_i + \boldsymbol{\gamma} \cdot \boldsymbol{z}_i \tag{19}$$

where

$$\begin{cases} \alpha = \frac{1}{\nu_p} \\ \beta = t_L \\ \gamma = \frac{1}{k} \end{cases}$$
(20)

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$$\begin{cases} x_i = d_i^o \\ y_i = m'_i \\ z_i = \frac{W_i}{M_i \cdot F_i \cdot n'} \end{cases}$$
(21)

In equation (19), parameters such as  $\alpha$ ,  $\beta$ ,  $\gamma$  are identified from regression analysis from the observed data set  $(t_i^T, x_i, y_i, z_i)$ .

#### Results

The average tortuosity factor was 1.52 (  $\delta_{n-1}=0.703$  ).

The results of multiple regression analysis are listed in Table 1. The t-value of  $\beta$  and  $\gamma$  were large enough to clear 1% significance. The parameters calculated from the these parameters are given in Table 2. Although the /-value of  $\alpha$  was low, the

Table 1 Results of multiple regression analysis

	a (s/m)	b (s/station)	g (s/kg)
Value	0.115	12.1	2.78
t-value	1.48	7.54**	3.83**
r2 = 0.67	9		

\*\* 1% significance

Table 2	Velocity,	Loss	time,	and	Loading	rate
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Velocity (v <sub>p</sub> )	Loss time (t <sub>1</sub> )	Loading rate (k)
31.5 km/h	12.1 s/station	0.361 kg/s

value of  $v_p$  which is derived from  $\alpha$ , does not contradict our intuitive expectation.

In Table 3, we tabulated the results of our actual observation of collection process. The observed value of the loading rate coincided with the observation. The observed value of velocity were somewhat smaller than the result of our analysis. However, considering the low t-value of the parameter, we are satisfied that the result was only 50% larger than the observed values.

Matsui also observed the actual collection activity in Uji city (Matsui, 1994). He obtained the loss time, the loading time, the driving velocity between the collection center and the collection area as well as the driving velocity in the collection area. The loss time of our results (12.Is/station) are extremely in good agreement with the average value (10 s/station) reported by Matsui (1994). The driving velocity of our results (31.5km/hour) is around 50% larger than those reported by Matsui for drive between the collection center and the collection area, and around 2.5 times as much as the velocity reported for the drive in the collection area.

The correlation between the observed data and the predicted from equation (8) is depicted in Fig-

City	Population	Area (km²)	Velocity	7 (km/h)	Loading rate (kg/s)	Waste
			Collection	Transport	, ,, ,, ,, ,, ,, ,, ,, ,, ,, ,, ,,	
A	73,000	35	15.7	24.0	0.40	bottle/can
В	213,000	87	18.2	20.6	0.54	bottle
С	213,000	87	12.9	19.1	0.49	can

Table 3 Observed value of velocity and loading rate

ure 3. Considering the simplicity of the model, the agreement is excellent.

The predicted number of trucks are correlated with the actual data in Figure 4. The number



Fig. 3 Correlation between the actual distance and the calculated distance



Fig. 4 Correlation between the actual number and the calculated number of trucks

of collection trucks was also in good agreement with the observation.

## Application

In order to examine the environmental and economic effects of selective collection of PCW, let us calculate the actual cases and see what this model tells.

We take up two cases; sorted collection of PET bottles and mixed collection of household waste. Since PET bottles are one of the most bulky PCW, the environmental and economic effects of sorted collection of PCW are expected to be most evident when compared to mixed collection of household waste.

The values of parameters we assumed are given in Table 4.

The unit transport distance (D/W) and the number of collection trucks per a waste (M/W) are drawn as functions of the number of collection stations (N) for different collection frequency (f) in Figures 5, 6, 7, and 8.

#### Unit transport distance

As is expected, D/W increased with increase of both N and f (Figures 5 and 6). However, D/W of PET bottle was larger than that of house-hold waste for 10 to 100 times depending on both N a n d f.

Parameter	Value	
Population	3,230,000	
Area	385 km <sup>2</sup>	
Number of the recycling center	5	
Waste discard unit		
Household waste	1.14kg/day/capita	
PET bottle	0.00313kg/day/capita	
Effective capacity of a collection truck		
Household waste	1,400kg/truck	
PET bottle	110 kg/truck	
Tortuosity factor	1.52	
Working days of the recycling center	313day/year	
Working hours of the recycling center	8 hours/day	

Table 4 The parameters we used for calculation of Figures 5, 6, 7 and 8



Fig. 5 Unit transport distance of sorted collection of PET bottle





### Number of collection trucks

The results of calculation are given in Figures 7 and 8. Qualitatively, the results resemble to those of unit transport distance. M/W of PET.bottle is much larger than that of the household waste for 5 to 500 times depending on both N and f.

## Discussion

### Tortuosity factor

In our study, the average tortuosity factor was 1.52 ( $\delta_{n-1}$ ,=0.703). One of the reason why the average tortuosity factor was larger than unity may



Fig. 7 Number of collection trucks per unit waste in the case of sorted collection of PET bottle



Fig. 8 Number of collection trucks per unit waste in the case of mixed collection of household waste

be that the location of recycling center (waste treatment center) is likely to be in the peripheral area; which will cause less collection efficiency than that of the grid city model.

### Model performance

From comparison of the model prediction and the actual data, we can conclude that, in average, the Grid City Model can predict the transport distance and the number of collection trucks. However, we should be careful to apply this model to a specific city, in such a case, we must accept somewhat large error shown in Figure 3 or 4. However, even in such a case, the Grid City Model has advantage over simple scenario approach because the Grid City Model can give the value of error of the p r e d i c t i o n.

### Unit transport distance

D/W of PET bottle was larger than that of household waste for 10 to 100 times depending on both N and f. This indicates that the environmental merit of recycling would be somewhat discounted by the increase of transportation.

The difference between PET and household waste is explained by two reasons; one is the difference of effective density (hereafter we refer this effect as "density effect") and the other is the difference of waste discard unit (hereafter we refer this effect as "scale effect").

The D/W is expressed as the sum of a constant term and a  $N_{\perp}^{1/2}$  term. It is evident that the effective density affect only the first constant term, therefor this term describes the "density effect". The fact that the "density effect" appear only in the transport between the recycling center and the collection area, and not in the transport in the collection area looks to be peculiar. This is because that the total transport distance in the collection area can be described only by the total waste discarded: under a give collection frequency, one can calculate the number of collection stations that should be collected in a day, this means that the waste discarded in these stations in a day should be collected in a day no matter how bulky the waste might be, and this requires that the collection truck should visit all the collection stations in the area. Therefor the transport distance in the collection area is determined from only location of the collection stations.

Similarly, the waste discard unit (u) only affect the second term in equation (9). The reason why we call this as "scale effect" is that this term expresses the scale merit of collection. Consider the extreme case that the amount of waste discarded in one collection station is equal to the effective capacity of the collection truck, it is evident that the transport distance in the collection area should be minimal value of null.

The balance of these two effects depends on cases. In general, if the waste discard unit (u) is large or the collection frequency (f) is small, D/W would be mostly determined by the "density effect", and D/W would be almost independent of the number of the collection stations (N) nor the collection frequency (f); this is the case of household waste collection (Figure 6).

On the contrary, if the waste discard unit (u) is small or the collection frequency (f) is large, the "scale effect" would dominate in equation (10), and D/W would increase with the increase of N and f; this is the case of sorted collection of PET

bottle (Figure 5). Please note that the asymptotic values in Figures 5 and 6 are determined by the "density effect".

### Number of collection trucks

M/W of PET bottle is much larger than that of the household waste for 5 to 500 times depending on both N and f. This means that the sorted collection of PET bottle requires much larger cost than the conventional household waste collection. It is evident that the increase in cost could be reduced by a simultaneous collection of several selected wastes; since this increases the waste discard unit (u).

The difference between PET and household waste can be similarly explained by the "density effect" and the "scale effect". The "density effect" of M/W is defined by the first from of the equation (14), and the "scale effect" of M/W is defined by the sum of the terms except the first terms in the equation (14).

In the case of M/W, since the "scale effect" includes a term proportional to N, M/W increases much faster with increase of N compared with the case of D/W (Figure 7).

## Conclusions

A model (Grid City Model) was developed, which can predict the transport distance and the number of collection trucks from data such as population, area, tortuosity factor, waste discharge unit, number of collection stations, size of a collection truck, and frequency of collection.

Trade-off relation between the service level of waste collection and environmental load, and

between the service level of waste collection and the number of collection trucks are explicitly described in the Grid City Model.

Both the unit transport distance (D/W) and the number of collection trucks per a unit waste (M/W) are described as a sum of the "density effect" and the "scale effect": the "density effect" depends on the effective capacity of the collection truck, and independent of the waste discard unit (u) nor the collection frequency (f); the "scale effect" depends on both the waste discard unit (u) and the collection frequency (f), and independent of the effective capacity of the collection truck.

Both D/W and M/W of sorted collection of PET bottle is much larger than those of the household waste in a typical case.

## A c k n o w l e d g m e n t

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家庭系廃棄物リサイクルの輸送モデル

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本論文では、家庭系から排出される包装廃棄物のリサイクルのための輸送モデル(Grid City Model)を提案する。このモデルによれば、輸送距離と必要車両台数を予測できる。必要なパラメー タは、人口、面積、廃棄物発生原単位、ステーション数、収集車容量、収集頻度である。収集のサービ ス水準と環境負荷、収集車台数の間のトレードオフ関係がこのモデルによって明示的に記述された。 廃棄物当りの輸送距離(D/W)と廃棄物当りの収集車台数(M/W)は密度効果と規模効果にわける ことができた。密度効果は、収集車の有効積載量のみで決まり、規模効果は、廃棄物発生原単位と、収 集頻度で決まることが示された。典型的なケースにおいてPETボトルの分別収集のD/WとM/W は、ともに家庭系廃棄物の混合収集に比較して遥かに大きかった。

キーワード:リサイクル、輸送、モデル、家庭系廃棄物、LCA

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